Roton Quantum Model - Mathematical Description

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1 Introduction

The **Roton Quantum Model** introduces a hierarchical structure built from fundamental energy units called **Twons**. This model defines energy levels through rotational dynamics, without explicitly relying on mass, and introduces unique, orientation-dependent forces between its structures. Unlike conventional systems, Rotons are self-sustaining and can split or merge naturally based on local energy conditions, potentially creating highly dynamic, self-organizing clusters over time.

2 The Twon - Base Energy Unit

We define the fundamental energy unit, e_0, called a **Twon**, which is assigned to a spherical volume V_0 of radius r_0:

$$V_0 = \frac{4}{3}\pi r_0^3$$

3 The Roton - Two Twons in Rotational Motion

At the next energy level, we pair two Twons to form a **Roton** with self-sustaining rotational energy:

$$e_1 = 2e_0\omega_0^2 r_0^2 = 2e_0\frac{v_0^2}{r_0^2}r_0^2 = 2e_0 v_0^2$$

where:

- e_0 is the energy of a single Twon,
- ω_0 is the angular velocity of the Roton,
- v_0 is the corresponding tangential speed.

4 Rotonal Forces

4.1 Attractive Rotonal Force F_2

The attractive force between two Rotons, e_{1_α} and e_{1_β}, is defined as:

$$F_2 = k_2 \frac{e_{1_\alpha} e_{1_\beta}}{d_2} \cos(\theta) \cdot G(r_{1_\alpha}, r_{1_\beta})$$

where:

- d_2 is the distance between the centers of the two Rotons,
- θ is the relative angle between their rotational axes,
- $G(r_{1_\alpha}, r_{1_\beta})$ is a Gaussian-like matching function that accounts for the difference in their effective radii

$$G(r_{1_\alpha}, r_{1_\beta}) = \exp\left(-\frac{(r_{1_\alpha} - r_{1_\beta})^2}{\sigma^2}\right)$$

This function ensures that Rotons with more similar radii have a stronger attractive force, encouraging hierarchical clustering of similarly sized energy structures. This choice reflects the idea that overlapping energy fields with similar scales naturally form more stable connections.

There is an **exception** or limit on the validity of this function based on the modelling. The attractive forces between Rotons only adhere to this formula, if the distance is bigger than the rotation radius of the Roton. If the distance is smaller, the attractive force might be different (not yet fully modelled). This is not modelled in the given function.

4.2 Repulsive Energy Force R_d

Each Roton radiates waves within the medium leading to an energy density field. This field has a mostly radial pattern (in a planar view), creating a **Rotonal repulsive Force** R_d that generally acts to push other Rotons away:

$$R_d = k_R \frac{e_{1_\alpha} e_{1_\beta}}{d_2^2} H(\theta)$$

where:

- k_R is a proportionality constant,
- $H(\theta)$ is an angular function that reduces the repulsive effect along the axis of rotation, such as:

$$H(\theta) = \cos^2(\theta)$$

This function reflects the reduced repulsion along the rotational axis, where energy density is weaker. We will currently eliminate this function though and define it as $H(theta) = 1$. This is due to the fact, that the attractive forces are already based on directional and angular differences.

4.3 System Dynamics and Stability

Given these competing forces, the overall interaction between two Rotons is:

$$F_{net} = F_2 - R_d$$

Over long distances, the attractive force F_2 dominates, encouraging clustering, while at short ranges, the repulsive pressure R_d prevents collapse into singular structures. Furthermore the attractive force fades away within the range of the roton radius. This dynamic creates a naturally fluctuating system where stable configurations are statistically preferred but not strictly enforced, allowing for the possibility of spontaneous symmetry breaking and complex, emergent behaviors.

5 Emergence and Self-Organization

As this system evolves, we expect the following behaviors:

- **Clustering and Fragmentation:** Rotons may cluster into larger, stable configurations, or split into smaller units if local conditions are unfavorable.
- **Self-Optimization:** Over time, a system of Rotons may settle into configurations that maximize attractive forces while minimizing repulsive pressures. Or in other words lead to positions/constellations which optimize (maximize) the energy density of the system. In a dynamic flow.
- **Fluctuations and Reorganization:** How does the system reach this point of optimal energy density? This is not a strict constraint opposed by the nature of physics. Over time though, the angular differences in the forces lead to an alignment of the rotational axis and speeds, triggered by local fluctuations which drive the system toward more energetically optimal states.

5.1 Preliminary Python Simulation (Optional)

A simple Python function to calculate forces for a set of Twons might look like:

```
import numpy as np

def force_between_rotons(e1_alpha, e1_beta, r_alpha, r_beta, d, theta, k2, kR, sigma)
   # Attractive force
   G = np.exp(-((r_alpha - r_beta)**2) / sigma**2)
   F2 = k2 * (e1_alpha * e1_beta) / d * np.cos(theta) * G

   # Detractive force
   Rd = kR * (e1_alpha * e1_beta) / d**2 * np.cos(theta)**2

   # Net force
   F_net = F2 - Rd
   return F_net
```

TODO: The formulas are not the ones used in the provided animations - need to be updated.

This function can be used to simulate the interactions between thousands of Rotons, allowing us to explore the emergent behaviors of this system on a larger scale.

6 Conclusion

The **Roton Quantum Model** offers a novel framework for understanding energy structures without explicit mass, focusing instead on rotational energy hierarchies and complex force interactions. The model's flexibility allows for spontaneous clustering, symmetry breaking, and the potential for higher-order structures, reflecting the dynamic, self-organizing nature of physical reality.